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The Wave Logic of Consciousness: A Hypothesis[1]

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A physical model is proposed for volitional decision making. It is postulated that consciousness reduces *doubt states* of the brain into labels by a quantum-mechanical measurement act of *free choice*. Elementary doubt states illustrate analogical encodement of information having "insufficient resolution" from a classical viewpoint. Measures of *certitude* (inner conviction) and doubt are formulated. "Adequate propositions" for nonclassical statements, e.g., Hamlet's soliloquy, are constructed. A role is proposed for the superposition principle in imagination and creativity. Experimental predictions are offered for positive and negative interference of doubts. Necessary criteria are made explicit for doubting sense information. *Wholeness of perception* is illustrated using irreducible, unitary representations of n-valued logics. The interpreted formalism includes nonclassical features of doubt, e.g., scalor representations for imprecise propositions and state changes due to self-reflection. The "liar paradox" is resolved. An internal origin is suggested for spinor dichotomies, e.g., "true-false" and "good-bad," analogous to particle production.

1. THREE TYPES OF DECISION MAKING

People have always distinguished between "choosing in order to" and "choosing because of"—say, because of one's sense of duty. However, many scientists believe that in some general sense all decisions are made "in order to." In the theory presented in this paper the following hypotheses concerning decision making will be assumed:

(1) There are decisions which result as (unique) deterministic consequences of "in order to" considerations. But the criteria and ultimate goals on which these considerations have rested are not necessarily deterministic.

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(2) There are acts of decision making which can be, from the point of view of methodology, conveniently treated as random, but which are in fact uniquely determined (they may be discovered, in principle, if more perfect research methods are used). Many biologists are convinced that it is just this mechanism of seeming randomness that constitutes the basis of what is called "free choice."

(3) On the other hand, there are acts of choice which are spontaneous by their nature. Their relative probabilities are represented by the corresponding probability amplitudes, which are considered in details below.

The term "relative probability of a choice" is used in the following sense. I believe that the act of decision making may be decomposed into two phenomena: (a) emergence of a wish, or the awareness of the necessity to make some choice; (b) the act of a *specific* choice, which in the simplest case will appear as an answer "Yes" or "No" to some question. The probability amplitude which I am introducing refers only to the stage (b), i.e., it describes the relative (conditional) probability of each specific choice assuming that some choice will be made necessarily—and not the full probability of the emergence of an answer of "Yes" or "No." We presuppose the spontaneity of a *specific* choice; and if this choice is spontaneous then, according to our theory, the distinction between stages (a) and (b) will be quite clear. This act of a spontaneous choice I will further call *free choice*.

A state of consciousness in which this or that result of choice making is manifested without a detectable cause (of just *this* result) I will call a *doubt state*.

It is easy to see that the volitional act of a free choice plays in this theory a role analogous to the role of the measurement act in quantum mechanics (with the important difference that the brain "measures" itself). Therefore, we postulate that the doubt states are quantum-mechanical states of the brain. But it is also possible that "the doubt field" is an independent entity which cannot be reduced to quantum-mechanical functions of brain particles. Consciousness is a system which observes itself and evaluates itself—being aware, at the same time, of doing so. The present physical theory does not have an apparatus to describe such systems. Thus it is not excluded that new variables ("doubt variables") may need to be introduced in order to describe such a system.

In clarifying the phenomenon of free choice, I must add that by a free choice I mean, strictly speaking, only a *formulation* of a specific choice, after which a corresponding action may or may not follow. But this formulation must somehow be fixed by consciousness as *a message*. This message is a phenomenon in an array or a list of other phenomena related to information transfer, and the brain will analyze it as it does other messages, "looking at itself from outside."

In a more general sense, by free choice we may mean the selection and fixation by consciousness of its own state which results from (or, in some particular cases, persists after) this very fixation. For according to the logic of the present theory, formulation of a choice generally changes the preceding state, forming a new state (in a manner not unlike a quantum-mechanical measurement).

It should be noted in this connection that a measurement in quantum mechanics changes the state and forms a new state only if the information obtained in the act of measuring is used for prediction or selection of particular cases in a following measurement. If the information is not used, then the measurement is nothing more than a usual act of interaction inside the system extended to include the detectors of interactions; there will be no reduction of the wave packet. Using the analogy: *measurement* corresponds to *free choice*; one can assert that a free choice does not change the state of consciousness if the corresponding formulation is immediately forgotten by the agent.

2. ELEMENTARY DOUBT STATE. NONCLASSICAL PROPOSITIONS

By its definition, an *elementary doubt state* contains information about the truth value of a proposition which in this context may be considered as elementary or atomic, in the terminology of classical logic. But one must bear in mind that in the wave logic not every atomic proposition is [restricted to, nor] expressed in the form of, a grammatically correct verbal [symbolic] message.

Let us consider the following example: suppose that an adult says to a child, pointing at a pack of wool,

$$\text{“This is a wolf.”} \qquad (1)$$

If the child has had experiences that enable him to attach meaning to the phrase "wolf," and if, furthermore, it is essential for him to make a decision as to whether it is a wolf, then he may find himself in one of the following three states:

A. He believes the adult. Then according to the present theory, with respect to proposition (1) he is in a state which is described by the spinor

$$\varphi_0^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \text{[proposition (1) is true]} \qquad (2)$$

B. He is certain that it is not a wolf. Then his state with respect to proposition (1) is the spinor

$$\varphi_0^0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \text{[proposition (1) is false]} \tag{3}$$

C. With the information he has he is unable to solve the dilemma; we may say that he *lacks necessary resolution*. Then he is in an elementary doubt state of the general type which is described by the spinor

$$\varphi = \begin{pmatrix} a \\ b \end{pmatrix}, \qquad |a|^2 + |b|^2 = 1 \tag{4}$$

with $a, b \neq 0, 1$. According to our main assumption, $|a|^2$ is the probability that upon being forced to make a decision in this state, the child will say to himself "Yes, it is a wolf" [i.e., proposition (1) is true]; $|b|^2$ is the probability of choosing the answer "No."

For the state (4) to exist it is essential that there exists a conflict: on one side, it is impossible to solve the dilemma, and on the other side, it is necessary to solve it and in doing so rely upon one's own "inner voice," and not, say, a generator of random numbers. There is also a possible situation in which a person is completely or partially indifferent to the problem, so that only some probabilities can be given regarding the doubt state of the person. This more general situation is described by a density matrix

$$\rho_k^i = \begin{pmatrix} \rho_1^1 & \rho_2^1 \\ \rho_1^2 & \rho_2^2 \end{pmatrix}, \qquad \rho_1^1 + \rho_2^2 = 1 \tag{5}$$

Here ρ_1^1 is the probability that the person will say "Yes," and ρ_2^2 is the probability of "No." In case of complete indifference to the problem

$$\rho = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \tag{6}$$

As is well known, state (5) is called a *mixed* state or a mixture in contrast with *pure* state (4). State (5) may also be thought of as describing an ensemble of persons who are in different pure states.

Let us return to the described conflict. When the child manifests his state (4) in a straightforward fashion, he uses not only words ("This—is it a wolf??") and not only intonation (which we tried to render here through a due number of question marks), but also facial expression and other ways and means which constitute a continuum [i.e., the inner automorphisms of the algebra of the observables]. This is referred to as a *nonclassical proposition* in the present theory. It can be formally described by an operator.

According to my paper presented at the 5th Congress on Philosophy, Methodology of Science and Mathematical Logic (Orlov, 1978a),[2] every atomic proposition of classical logic can be represented by a diagonal operator— the third component of the Pauli algebra σ_3:

$$\sigma_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \qquad \sigma_3^2=1, \qquad \sigma_3^\dagger=\sigma_3 \tag{7}$$

which is applied to spinor truth functions, representing the state of doubt of an elementary proposition, in accordance with the usual rules of matrix multiplication. $\sigma^\dagger$ is conjugate to σ. The diagonal terms in (7) represent possible eigenvalues: $+1$ (proposition σ_3 is true) and -1 ([proposition σ_3 is] false). States (2) and (3) are classical doubt states, corresponding to no doubt at all. When a set of independent atomic propositions is considered, operators and their corresponding functions are assigned sequential numbers (propositions in classical logic are denumerable). It is easy to see that when the classical approximation is valid

$$\sigma_3\varphi_0^1=\varphi_0^1, \qquad \sigma_3\varphi_0^0=-\varphi_0^0, \qquad \sigma_3\varphi_0^\lambda=(2\lambda-1)\varphi_0^\lambda, \qquad \lambda=0,1 \tag{8}$$

(Index 0 marks the classical character of the functions, λ is an occupation number.)

Doubt state (4) is recorded in the reference system (we might say "*from the viewpoint*") of a classical observer, whose own propositions are of the diagonal form (7), and the doubt states are of the form (2) or (3). In their verbal expression the propositions of a classical observer are propositions of the traditional classical logic.

The norm of the average value of the operator σ_3 in a given state n determines the degree of *certitude* (*inner conviction*)

$$y_n=|\langle\sigma_3\rangle_n|\equiv|\varphi_n^\dagger\sigma_3\varphi_n|\equiv\big||a_n|^2-|b_n|^2\big| \tag{9}$$

$$0\leqslant y_n\leqslant 1$$

The quantity

$$c_n=1-y_n \tag{10}$$

determines the amount of doubt.

[2]See also Orlov (1978b) (*Translator's note*).

Let us find the operator expression for a nonclassical proposition which is an *adequate manifestation* of the doubt state (4) ("*self-expression*"). State (4) should be an eigenstate for this operator with the eigenvalue $+1$. (The *adequate proposition* is assertory; it combines both a usual proposition of logic, which may be either true or false, and a proposition by an observer about the truthfulness of the proposition, the observer being the speaker himself.) Because of this eigenvalue we shall henceforth denote a doubt state without a subindex. The superindex of the state φ^1 would indicate that the eigenvalue is $+1$. Similarly the state φ^0, which is orthogonal to the state φ^1, must correspond to the eigenvalue -1 of the adequate operator. In addition, the operator that we are looking for, like all operators of propositions, must be self-conjugate (Hermitian) and its square must be unity (two-valued logic):

$$\sigma\varphi^1 = \varphi^1, \qquad \sigma\varphi^0 = -\varphi^0, \qquad (\varphi^0)^\dagger\varphi^1 = 1, \qquad \sigma^2 = 1, \qquad \sigma^\dagger = \sigma \tag{11}$$

Here, as usual, if

$$\varphi^0 = \begin{pmatrix} c \\ d \end{pmatrix}$$

then

$$(\varphi^0)^\dagger = (c^*, d^*), \qquad (\varphi^0)^\dagger\varphi^1 = ac^* + bd^*$$

The equations (11) define the operator sought except for an arbitrary phase factor by which the function φ^0 may be multiplied if the function φ^1 is given. However, it is more convenient to find the adequate proposition σ in a different manner.

States φ^1 and φ^0 may be regarded as the result of a linear transformation of classical states φ_0^1 and φ_0^0, i.e.,

$$\varphi^1 = S\varphi_0^1, \qquad \varphi^0 = S\varphi_0^0, \qquad S^\dagger = S^{-1}, \qquad |S| = 1 \tag{12}$$

where S is a 2×2 matrix, $S^\dagger$ and S^{-1} are conjugated and inverse matrices, $|S|$ is the determinant of S. Let

$$S = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \tag{13}$$

We introduce new variables by these equations:

$$a = A\exp(i\alpha), \qquad b = B\exp(i\beta), \qquad c = C\exp(i\gamma), \qquad d = D\exp(i\delta)$$

$$B = (1 - A^2)^{1/2}, \qquad D = A, \qquad C = B, \qquad \alpha + \delta = 2k\pi,$$

$$\beta + \gamma = (2k+1)\pi, \qquad k = 0, \pm 1, \pm 2, \ldots$$

According to a well-known formula

$$\sigma = S\sigma_3 S^\dagger \tag{14}$$

Choosing $k=0$, we obtain $\delta = -\alpha$, $\gamma = \pi - \beta$, and finally

$$\varphi^1 = \begin{pmatrix} Ae^{i\alpha} \\ Be^{i\beta} \end{pmatrix}, \qquad \varphi^0 = \begin{pmatrix} -Be^{-i\beta} \\ Ae^{-i\alpha} \end{pmatrix}, \qquad S = \begin{pmatrix} Ae^{i\alpha} & -Be^{-i\beta} \\ Be^{i\beta} & Ae^{-i\alpha} \end{pmatrix} \tag{15}$$

$$\sigma = \begin{pmatrix} A^2 - B^2 & 2ABe^{i(\alpha-\beta)} \\ 2ABe^{-i(\alpha-\beta)} & B^2 - A^2 \end{pmatrix} \tag{16}$$

Matrix S offers a new approach, or *new criteria for truth* with respect to an elementary proposition for a person (*characterized by this matrix*) who is different from a classical observer. For such a person, the quantities

$$y = |A^2 - B^2| = |2A^2 - 1|, \qquad c = \begin{cases} 2A^2 & \text{if } A^2 < \frac{1}{2} \\ 2B^2 & \text{if } A^2 \geqslant \frac{1}{2} \end{cases} \tag{17}$$

show a decline in certitude and growth of doubt when A is different from both 1 and 0. As to the phase factors, their meaning may be discovered in experiments on *the interference of doubts* (see below).

One familiar example of a nonclassical proposition, which has been fairly accurately expressed in words and in intonation, is Hamlet's proposition:

$$\text{“To be or not to be?”} = \begin{pmatrix} 0 & e^{i\zeta} \\ e^{-i\zeta} & 0 \end{pmatrix} \tag{18}$$

which is an adequate manifestation of his doubt state

$$\varphi^1 = e^{i\alpha}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{-i\zeta} \end{pmatrix} \tag{19}$$

Here α is an arbitrary phase.

As we have already noted above, an adequate proposition, being an act of "self-expression," does not change the state of the speaker. The speaker displays a minimum of volition; to be more precise, his *will* is not directed towards changing his state. Nonadequate propositions will change the state and are, in this sense, acts of violence over oneself. When the child says to himself

"This is *not a* wolf" (20)

and laughs cheerfully, there will be a reduction of his state with respect to proposition (1):

$$\begin{pmatrix} a \\ b \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{21}$$

Generalizing, we can say that after any statement the state of the speaker [will be *projected* upon and thereby] become an eigenstate of the stated proposition with the eigenvalue $+1$.

Now the question arises of statements which are known to be insincere. For instance, consider how proposition (1) is related to the inner state of the adult who is stating it. If the statement was accompanied by some signs that the speaker was kidding, then it would be, on the whole, an adequate manifestation of the adult's nonclassical proposition. Suppose there were no signs of that sort. Then, according to the logic of our theory, the state of the adult before the statement could not be (3) [or (20)], for in this [orthogonal] state the probability to make choice (2) is zero. The state of his inner conviction should have been a *superposition of* the two states (3) and (2):

$$\varphi_{\text{before statement}} = u\begin{pmatrix} 0 \\ 1 \end{pmatrix} + v\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} v \\ u \end{pmatrix} \tag{22}$$

where, possibly, $|v|^2 \ll |u|^2$ but still $|v|^2 \neq 0$. The admixture of state (2) ("[It is true that?] this is a wolf.") in the consciousness of the adult arose perhaps as a result of combining impressions of a pack of wool with impressions evoked from memory of a wolf. In other words, because of the work of *imagination* in the mind of the adult; two, and not one, items appeared: "wool" and "wolf" (so that the actual function is still more complicated than (22)—see next section). At any rate, the probability amplitude v of "This is a wolf" was not zero before proposition (1) was stated. This is why the adult could say (1)—with conditional probability $|v|^2$. As the result of stating (1) the adult continued to see a wolf in the pack of wool during some time, which would be the longer the more successful the adult was in exercising his will and his imagination. During this time he embodied state (2), for at the moment of stating (1) the following reduction took place:

$$\varphi_{\text{before statement}} = \begin{pmatrix} v \\ u \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \varphi_{\text{after statement}}$$

Later, probably, his state drifted in the direction (2)→(3) under the influence of the information from sense organs, slackening of imagination, etc.

The described mechanism explains the appearance of statements "known to be false" (from the viewpoint of a classical observer), and gives a key to the understanding of *creative thinking*, when a person states or depicts "what in fact does not exist." According to our approach, the person potentially "sees" several versions simultaneously without completely realizing any of them, and then one version "pops up" (materializes) as the result of a free choice. This version will be fixated in the brain structure for some time, during which it will be treated as any other piece of information.

In analyzing proposition (1) an important assumption of the theory was used: linearity and the *superposition principle* (in intervals between acts of choice!). It is assumed that the doubt states induced by different external and internal sources of information sum up linearly until the *will* interferes and changes the state by an act of free choice. It would not be unreasonable to suppose that in the intervals the state functions are developing as if they described a certain free field. This process (which could be called "wave thinking") would also obey some linear equations.

From the principle of superposition of probability amplitudes it follows that some effects of "interference of doubts" must be observable. The phenomenon of being blind to obvious facts might be properly explained by destructive interference, while "seeing" nonexistent things might be the result of constructive interference.

Now let us consider the problem of measuring the probability amplitude in the case of an elementary doubt state function.

Generally, the measurement should be performed either in an ensemble of *identical* individuals, or for a specific individual provided that he does not remember the circumstances nor the results of each consecutive act of measurement. An ensemble of (in a sense) "identical" individuals may be modeled, e.g., by mass hypnosis, if it does not come in conflict with the nature of the measurement. In other experiments, "identical" individuals may be approximated by real individuals.

In measurements of this kind, $|a|^2$ is the relative frequency of the answer "Yes," and $|b|^2$ is that of the answer "No." The measurement of the phases may be based on the interference of doubts; of course, only the phase differences will be measured. The layout of the phase measurement in the child and adult situation mentioned above might be, for instance, as follows.

Suppose that there are *two* different objects, 1 and 2, which are *separately* presented to children as "wolves." Each tested child sees only one object. The set of answers gives the experimenter an estimate of $|a_1|^2$ (for the first object) and $|a_2|^2$ (for the second object). After a while, the children (presumably other individuals who make up a more or less identical ensemble) are shown both objects simultaneously and told that "there is a wolf." Again, the relative frequency of the answer "Yes" is measured.

According to our theory, the doubt state of the tested child in the second experiment is

$$\begin{pmatrix} a_3 \\ b_3 \end{pmatrix} = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} + \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}\right], \qquad |a_1|^2 + |b_1|^2 = |a_2|^2 + |b_2|^2 = 1 \quad (23)$$

from which we obtain

$$A_3 \equiv |a_3|^2 = \tfrac{1}{2}|a_1 + a_2|^2 = \tfrac{1}{2}\left[A_1^2 + A_2^2 + 2A_1A_2\cos(\alpha_1 - \alpha_2)\right]$$
$$a_k = A_k \exp(i\alpha_k) \quad (24)$$

Using A_1^2, A_2^2, and A_3^2 obtained in the experiment, we find $\alpha_1 - \alpha_2$. Analogously, we find $\beta_1 - \beta_2$.

The destructive interference, if it takes place, will manifest itself in the experiment as the inequality

$$A_3^2 < (A_1^2 + A_2^2)/2 \quad (25)$$

3. A DOUBT IN THE CORRECTNESS OF THE INTERPRETATION OF SENSE INFORMATION

It seems plausible that the earliest type of doubt to appear in the evolution of life (which is also the most important one) is doubt that the information from the sense organs is interpreted correctly. For an individual to experience this type of doubt, it is necessary that:

(a) there are information signals;
(b) these signals evoke at least two competing associations in the individual's imagination, and the individual is not indifferent to at least one of them;
(c) it is necessary to make a decision as to whether one of the associations (that one to which the individual is not indifferent) is correct;
(d) the resolution is inadequate to choose between the competing associations on the basis of computation;
(e) the individual is in such a state that he cannot entrust the decision making to a generator of random acts (i.e., he is convinced that he must do it *himself*).

It is important that, although according to (d) the individual in the *given* situation cannot make a choice between the alternatives, these alternatives, according to (b) can exist *separately* in his imagination. In other words, somehow, in his imagination, these alternative associations, e.g., "wolf" and "wool," must be *labeled*, or "named"—in the broad sense of the word. The existence of such labels is necessary for the appearance of a doubt state, which is, essentially the probability amplitude of *naming*.

Thus even in the case when the individual concerned does not seem to know how to manipulate with words, the investigator is justified in applying the apparatus of classical propositional logic, provided that the individual knows, in principle, how to distinguish one group of associations from the other. The elementary willed act of free choice is manifested here by putting the signals into correspondence with one of the groups of associations which appeared in his imagination; or, on the contrary, by denying such a correspondence for a significant label. If all the other labels are important to the extent that they demonstrate the existence of alternatives, then the individual's state just before the choice will be of the form (4).

Consider now the more complicated case when it is important for the individual to recognize *two* alternatives simultaneously. For example, suppose that a musician must recognize two notes with frequencies ω_1 and ω_2, ($\omega_1 \approx \omega_2$) in a complex sound, i.e., to distinguish which of the four cases is taking place: only ω_1 is present; only ω_2; both ω_1 and ω_2; or neither ω_1 nor ω_2. If he does not have the requisite resolution, then his state will be described by the superposition

$$F = a\varphi_{01}^{1}\varphi_{02}^{1} + b\varphi_{01}^{1}\varphi_{02}^{0} + c\varphi_{01}^{0}\varphi_{02}^{1} + d\varphi_{01}^{0}\varphi_{02}^{0} \tag{26}$$

where φ_0^1 and φ_0^2 are defined by (2) and (3), and the second subscript refers to the frequencies ω_1 and ω_2. $\varphi_{0k}^{\lambda_k}$ is the eigenfunction of the proposition

$$(\sigma_3)_k \equiv \text{"there is } \omega_k \text{ here"}, \qquad (\sigma_3)_k \varphi_{0k}^{\lambda_k} = (2\lambda_k - 1)\varphi_{0k}^{\lambda_k} \tag{27}$$

The product of the functions here is to be understood as a tensor product.

The state (26) is one of the four orthonormal states F^k, in which a given individual may be, in general, a given individual in a given situation is characterized by a transformation matrix S from a classical observer to the specific individual. The dependence of S on the situation can be seen from the fact that if somebody played notes ω_1 and ω_2 to the musician separately, he would be able to recognize them: we assumed that there are necessary labels in his consciousness. In our case, the transformation S can be represented by a 4×4 matrix operating on columns F_0^k, which are in a one-to-one correspondence with the functions of the classical observer

$\varphi_{01}^{1}\varphi_{02}^{1}, \varphi_{01}^{1}\varphi_{02}^{0}, \varphi_{01}^{0}\varphi_{02}^{1}, \varphi_{01}^{0}\varphi_{02}^{0}$:

$$F_0^1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad F_0^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad F_0^3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix},$$

$$F_0^4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad F = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}, \quad F^k = SF_0^k \tag{28}$$

As an example let the musician be in the state $F^2 = SF_0^k$. The classical proposition adequate to the classical state F_0^2 is

$$A_0 = (\sigma_3)_1 \& \neg(\sigma_3)_2 = \tfrac{1}{2}\left[(\sigma_3)_1 - (\sigma_3)_2 - (\sigma_3)_1(\sigma_3)_2 - 1\right] \tag{29}$$

(i.e. "there is ω_1 here, but no ω_2"). Operator A is equivalent to unity only when applied to F_0^2; otherwise it is (-1); in the representation (28) it is

$$A_0 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

From these considerations we obtain the adequate nonclassical proposition A of the musician in his state:

$$A = SA_0S^{\dagger} \tag{30}$$

This example shows how to find adequate propositions of arbitrary states (if the transformation matrix S is known, of course).

In the general case S *cannot* be reduced to the Cartesian product $S_1 \times S_2 \times \cdots$ of transformations each of which refers to one elementary proposition $(\sigma_3)_k$ only. This important fact causes the perception of information by the individual *to depend on the wholeness of the perception*. This phenomenon, known qualitatively to everybody from personal experience, is explained in the wave logic as follows.

Let the individual perceive the proposition (29) of the classical observer. Then two cases are possible:

(1) If the individual makes his choice immediately after *each* of the elementary propositions $(\sigma_3)_1$ and $(\sigma_3)_2$, he does it on the basis of some elementary states $\varphi_1^{\lambda_1}$ and $\varphi_2^{\lambda_2}$, which are related to the states of the classical observer by elementary transformations $\varphi_1^{\lambda_1} = S_1\varphi_{01}^{\lambda_1}$, $\varphi_2^{\lambda_2} = S_2\varphi_{02}^{\lambda_2}$. In this case

$S = S_1 \times S_2$. The connectives & and $\neg$ are understood identically by the classical observer and by the individual—separately, so to say, from propositions $(\sigma_3)_1$ and $(\sigma_3)_2$.

(2) If the individual's will to choose between alternatives is "turned off" until the whole complex sentence is perceived, his final state will be determined by a matrix which has a general form that is not a Cartesian product but a tensor product of elementary matrices. The connectives &, $\neg$, etc. are not separated in perception from elementary sentences, which in this case are not separated from one another either. Therefore, each complex sentence, which is perceived as a whole, must have its own transformation matrix S. In the general case, this produces a very complicated picture of the individual's perception of information, which changes with each act of choice.

The picture is still more complicated by the existence of *imprecise propositions*. Suppose, e.g., that the classical observer uses an imprecise word "ω_{12}," which may equally denote either ω_1 or ω_2. Then proposition

$$\Omega_3 = \text{"here is } \omega_{12}\text{"} \tag{31}$$

can be expressed by $(\sigma_3)_1$ and $(\sigma_3)_2$ (referring to notes ω_1 and ω_2) in the form:

$$\Omega_3 = \sum_{k=1}^{2} W_k(\sigma_3)_k = \tfrac{1}{2}\left[(\sigma_3)_1 + (\sigma_3)_2\right], \qquad W_1 = W_2 = \tfrac{1}{2} \tag{32}$$

Here W_k is interpreted as the probability that when asked "How do you understand the word ω_{12}?" an observer from a certain set of observers will answer: "ω_{12} is ω_k."

Every imprecise proposition (and, in fact, all propositions are imprecise) can be defined in our theory in an analogous manner. For instance, the proposition

$$\Omega_n = \text{"This is a young man"} \tag{33}$$

can be represented in the form

$$\Omega_n = \sum_{k=1}^{N} W_k(\sigma_3)_k, \qquad \sum_{k=1}^{N} W_k = 1 \tag{34}$$

where

$$(\sigma_3)_k \equiv \text{"This is a man of } k \text{ years,"}\ k \leqslant N \tag{35}$$

and W_k is the relative frequency of the answer (35) in a poll.

Returning to Ω_3 we see that it is a proposition of a three-valued logic, since the operator Ω_3, according to (32), has the eigenvalues 1, 0, −1. Thus the emergence of n-valued logics (with $n \neq 2$) is related in our theory to the introduction of imprecise propositions. The states of the classical observer

$$F_0^{\lambda_1,\lambda_2,\ldots,\lambda_k} = \prod_{i=1}^{k} \varphi_{0i}^{\lambda_i} \equiv \varphi_{01}^{\lambda_1}\varphi_{02}^{\lambda_2}\cdots\varphi_{0k}^{\lambda_k} \tag{36}$$

(where here k is the number of elementary propositions that are significant in a given situation) are eigenstates for imprecise propositions of the type (34); but the eigenvalues corresponding to different variants of the set $\lambda_1,\ldots,\lambda_k$, now are not always equal to plus or minus unity.

It should be noted that imprecise propositions of the type (34) refer to concepts that correspond to quantities which are in principle measurable.

Let us now consider an interesting special case when the musician from the above-mentioned example is described by the following matrix:

$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{37}$$

The operator

$$\Omega_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{38}$$

applicable to functions (28), has "in the musician's language" the same form

$$\Omega_3 = S\Omega_3 S^{\dagger} \tag{39}$$

as "in the language of the classical observer," but the eigenfunctions of this proposition are now

$$\Phi_0^1 \equiv F^1 = \varphi_{01}^1\varphi_{02}^1, \qquad \Omega_3\Phi_0^1 = \Phi_0^1 \tag{40}$$

$$\Phi_0^2 \equiv F^2 = \frac{1}{\sqrt{2}}\left(\varphi_{01}^1\varphi_{02}^0 + \varphi_{01}^0\varphi_{02}^1\right), \qquad \Omega_3\Phi_0^2 = 0 \tag{41}$$

$$\Phi_0^3 \equiv F^4 = \varphi_{01}^0\varphi_{02}^0, \qquad \Omega_3\Phi_0^3 = -\Phi_0^3 \tag{42}$$

$$\Psi \equiv F^3 = \frac{1}{\sqrt{2}}\left(\varphi_{01}^1\varphi_{02}^0 - \varphi_{01}^0\varphi_{02}^1\right), \qquad [\Omega_3\Psi = 0] \tag{43}$$

$$F^k = SF_0^k \tag{44}$$

where we are redefining the functions F^k, the meaning of which will become evident from the following considerations.

Let us assume that the hypothesis that "the truth space" is characterized by rotational symmetry [as made in Orlov (1978a)] is true for the logic of consciousness. Then the functions Φ_0^k, $k=1,2,3$, are transformed by rotations according to an irreducible vector representation of the rotation group, just as the elementary states are transformed according to a spinor representation. Thus we are dealing with a class of individuals obeying (with respect to the proposition Ω_3) a three-valued logic and characterized by two types of doubt simultaneously. First, there is the doubt from the viewpoint of a usual classical observer, which is contained in the pure doubt state $\Phi_0^2 F^2$. Its magnitude can be found from a formula analogous to (10):

$$c_n = 1 - y_n, \qquad y_n = |\langle(\sigma_3)_n\rangle|, \qquad (y_n)_{F^1} = (y_n)_{F^4} = 1$$
$$(y_n)_{F^2} = 0 \tag{45}$$

(The subscript F denotes the state to which the computed quantities refer.) Second, there is the additional doubt of "the three-valued-logic individual" with respect to "the *classical* three-valued-logic individual," meaning by the latter an individual with the possible states Φ_0^k, $k=1,2,3$. The state of "the three-valued-logic individual" can be described (in the representation of "the classical three-valued-logic individual") by a column

$$\Phi = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, \qquad |a|^2 + |b|^2 + |c|^2 = 1 \tag{46}$$

or $\Phi = a\Phi_0^1 + b\Phi_0^2 + c\Phi_0^3$, while the three states of "the classical three-valued-logic individual" and the proposition Ω_3 in this representation have the form

$$\Phi_0^1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad \Phi_0^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \qquad \Phi_0^3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \qquad \Omega_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{47}$$

The three possible states Φ^k of the nonclassical "three-valued-logic individual", and the proposition Ω in "his language" are

$$\Phi^k = S\Phi_0^k, \qquad \Omega = S\Omega_3 S^\dagger \tag{48}$$

The magnitude of the additional doubt is determined by the formulas

$$c'_n = 1 - y'_n, \qquad y'_n = |\langle \Omega_n \rangle_\Phi|, \qquad y'_n \leq 1 \tag{49}$$

Assuming the above-mentioned symmetry, the state Ψ described by the scalar representation of the rotation group has a special significance. All the individuals in this state form a special class of people who have doubts only with respect to the propositions $(\sigma_3)_k$, i.e., they are unable to distinguish between ω_1 and ω_2. They do not have, and intrinsically cannot have, any doubts with respect to the proposition Ω_3, since they exclude the possibility of a simultaneous presence (or the negation of presence) of ω_1 and ω_2 in their interpretation of ω_{12}.

Thus imprecise propositions introduce additional complications. One should also take into account the complications created by the classical effects of random noises, and the effects of indifference, which make it necessary to use a density matrix.

4. THE DOUBTS RELATED TO SELF-EVALUATION

Human consciousness distinguishes itself from other systems known to science by its capability of self-evaluation. Each act of self-evaluation changes the state of the individual. The person who has said to himself: "I am bad" is no longer the same person he was before this statement. These well-known effects of "self-reflection" provide one of the arguments for the wave logic of consciousness because this logic includes the principle of forming a state by stating a proposition.

The domain of self-evaluations requires an introduction of its own variables independent of those variables which are related to external information. We should certainly include among the spinor doubt variables those such as "true–false" and "good–bad." Let us consider the proposition

$$\tau = \text{“}X \text{ is a liar”} \tag{50}$$

with the constraint that a liar can pronounce only a lie, and a nonliar tells only truth. Proposition τ may be considered classical if it is stated by a person distinct from X. But if it is stated by X, it is not a classical proposition any more. In such a situation, there is no way to establish the truthfulness of τ. From this example we see that a logic which allows (50) as an admissible proposition must, generally speaking, consider the notion of "true–false" independently of the notion of a correct recognition/interpretation.

The wave logic does not discard paradoxes of the type (50). The peculiarity of τ is only in the fact that if it is stated by X (let us denote τ as

τ_X in this case), then there is no actual individual for whom operator τ_X could be diagonal; i.e., τ_X is nonclassical for all observers. According to its definition, τ_X is "equally true and false." Therefore the form of the operator is identical to (18). Proposition τ_X must always be considered adequate to the state of the individual, which is described by a function of the type (19).

Similar considerations may be introduced with respect to the sentence

$$\rho = \text{"}X\text{ is a wicked man"} \tag{51}$$

If stated by X himself (i.e., $\rho = \rho_X$), such a sentence, for many of the observers, already cannot be just true or just false: if X is criticizing himself, he is not wicked to the end. Accordingly, the sentence (51) is somewhat nonclassical. Some will even find this sentence false solely because it is stated by X himself. This means that there are internal criteria for deciding "good or bad," *independent* of the external signals.

Neither variable pair "good–bad" nor "true–false" is amenable to an objective measurement. The variable "good–bad" cannot be considered equivalent to the variable "pleasant–unpleasant," which has a clearly discernable physiological basis. It is well known that human life abounds with conflicts of the type "pleasant but evil." Formally, they are describable in the same manner as the "two-atom" conflicts of recognition considered in the preceding section.

What are the origins of these variables—unmeasurable, although extremely important for the human spirit—of the type "true–false" and "good–bad"? I think that they are invented, emerging together with the corresponding state functions at the moment of invention. It resembles the emergence of particles from the vacuum, together with their wave functions. Extending the analogy, I am ready to suppose the existence of a "logical vacuum," filled with "a proposition field," which is in the "ground" or "zero-point" state. In other words, all the variables of consciousness—including those already invented, and those to appear in the future—exist potentially.

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